

**DEVELOPING ROBUST FFOPID CONTROLLERS
BASED ON FUZZY SET POINT WEIGHTING ALGORITHM**

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Abstract

Introduction of a novel structure controller which is called Fuzzy FOPID (FFOPID) controller is presented in this paper. The FOPID controller is combined with a fuzzy logic tuning system based on Fuzzy Set-point Weighting (FSW) algorithm. The proposed FFOPID is evaluated on different types of systems to demonstrate its effectiveness. In order to optimize the controller performance over both the time-domain and frequency-domain, a mixed H_2/H_∞ cost function is used. Comparisons are made with an FOPID and a PID controller from viewpoints of transient response characteristics. The simulation results clearly demonstrate the superiority of the proposed controller among the others.

2000 Mathematics Subject Classification: 26A33; 03E72, 26E50, 93C42, 54A40, 34K35, 93B36, 65K10

Key Words and Phrases: fuzzy tuning method, fractional order controllers, PID controllers, Fuzzy Set-point Weighting (FSW) algorithm, optimization, time response

1. Introduction

The Proportional-Integral-Derivational (PID) controllers have been an appropriate choice in many practical plants since 50 years ago. They have significant advantages such as simplicity, easy tuning, compatibility to practical processes, and so on. Despite of the advantages, it is proved that the PID controllers are effective for processes whose dynamic can be modeled up to second-order systems. However, real systems have sophisticated characteristic such as high-order dynamic, nonlinearity, delays, etc and also affect by noise, disturbance, and variation in environment conditions which cause uncertainty in model structure. To promote the PID capability, many approaches have been developed. Fuzzy inference system and fractional order controller are two main concerns of this paper. In recent years, the increasing number of studies related with the application of fractional controllers in many areas of science and engineering is remarkable, see e.g. [1]-[4]. A fractional order PID controller extends the conventional PID controllers by extending the ordinary differential equations to fractional differential ones. An FOPID controller is characterized by five parameters: the proportional gain, the integral gain, the derivative gain, the integrating order and the derivative order. Increasing the parameters for tuning can be led to more flexibility in order to reach high performance controllers. Several analytical ways were introduced to tune FOPID, see [5]-[11].

The purpose of this paper is to improve the performance of FOPID controllers by using experimental rules (intelligence) based on fuzzy logic. By the use of a fuzzy set point weighing (FSW) method, we obtain considerable improvement of the performance and robustness of the controller. This approach allows the values of the FOPID coefficients to vary on-line during transient disturbance occurring on the system.

On the other hand, a cost function based on both time and frequency domain, is used here in order to optimize the performance of the proposed Fuzzy FOPID controller. Particle Swarm Optimization (PSO) shows its well-behaved performance in optimizing complex and nonlinear functions in real word. In this paper, this approach is used in order to find optimal values for fuzzy system. The simulation results demonstrate great advantages of Fuzzy FOPID (FFOPID) based on FSW tuning method over present FOPID controllers.

This paper organizes as follows. PID tuning with fuzzy methods is reviewed in Section 2. In Section 3, we briefly introduce fractional order PID controllers. In Section 4 fuzzy tuning of FOPID controllers by the

means of FSW method is introduced. Sections 5 and 6, conclude this paper with the results of the proposed algorithm.

2. PID tuning with fuzzy methods

It was shown that the use of fuzzy system is an appropriate alternative to increase PID controller capabilities to improve system performance for a wide range of plants, since it allows making use of operator's experience and therefore adds some sort of intelligence to the automatic control. In this way, several fuzzy PID controllers have been developed, see [12]-[16]. Tzafestas and Papanikolopoulos proposed Incremental Fuzzy Expert PID control (IFE) method to scale the values of the three controller parameters (initially determined by the Ziegler-Nichols formula), [12]. During the IFE method, current values of the proportional, integral and derivative gains are increased or decreased by means of a fuzzy inference system, according to:

$$\begin{aligned} P &= P + CV\{e(t), \dot{e}(t)\} \times k_1 \\ I &= I + CV\{e(t), \dot{e}(t)\} \times k_2 \\ D &= D + CV\{e(t), \dot{e}(t)\} \times k_3, \end{aligned} \quad (2.1)$$

where the basic tuning is the Ziegler-Nichols one, $CV\{e(t), \dot{e}(t)\}$ is the output of the fuzzy inference system, based on the Macvicar-Whelan fuzzy rule matrix; k_1 , k_2 and k_3 are constant parameters that determine the range of variation of each term. The Fuzzy Self tuning of a Single Parameter (SSP) method devised by He et al., consists of a parameter the Ziegler-Nichols formula by means of a single parameter a , then using an online fuzzy inference system to self-tune the parameter [13]. In this way, the three PID parameters can be expressed as

$$\begin{aligned} k_p &= 1.2\alpha(t)k_u \\ T_i &= 0.75 \frac{1}{1+\alpha(t)}t_u \\ T_d &= 0.25T_i, \end{aligned} \quad (2.2)$$

where k_u and t_u are the ultimate gain and the period of the process, respectively. The value of $\alpha(t)$ comes from a recursive algorithm. According to the methodology proposed by Zhao et al. in [14], which is called Fuzzy Gain Scheduling (FGS), the three current PID parameters are determined as follows:

$$\begin{aligned} k_p &= (k_{p,\max} - k_{p,\min})k'_p + k_{p,\min} \\ k_d &= (k_{d,\max} - k_{d,\min})k'_d + k_{d,\min} \\ k_i &= k_p^2/(\alpha k_d), \end{aligned} \quad (2.3)$$

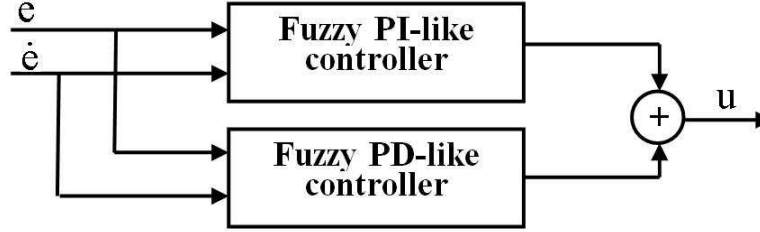


Figure 1: Fuzzy PID-like controller scheme.

where k'_p , k'_d and a are determined by means of a fuzzy mechanism and $k_{p,max}$, $k_{d,min}$, $k_{p,min}$ and $k_{d,max}$ are adopted to normalized the values of k_p and k_d into the range between zero and one. These constants are determined by the following rules of thumb:

$$\begin{aligned} k_{p,min} &= 0.32k_u, k_{p,max} = 0.6k_u \\ k_{d,min} &= 0.08k_u t_u, k_{d,max} = 0.15k_u t_u. \end{aligned}$$

In fuzzy PID-like controller scheme, the control variable is determined directly by means of a fuzzy inference system. The control variable is the sum of the outputs of a fuzzy PI-like controller and a fuzzy PD-like controller in Fig. 1.

This allows keeping a low number of rules without decreasing the performances. It turns out that two scaling factors for the two inputs and two for the two outputs of the PI and PD-like controller have to be selected by the user. Typical rules of thumb regarding the fuzzy controllers can be adopted in this case. In the last of 1990s, Visioli [15] proposed Fuzzy Set-point weighting (FSW) method for fuzzy PID tuning. In this scheme, the control variable is defined by

$$u(t) = k_p e_p(t) + k_d \frac{de(t)}{dt} + k_i \int_0^t e(\tau) d\tau, \quad (2.4)$$

where $e(t)$ is system error; $u(t)$ is control variable; k_p , k_i , and k_d are proportional, integral and derivative gains, respectively; y_{sp} and y are reference input and output, respectively, and $e_p(t) = by_{sp} - y(t)$ for $b < 1$. In this way, a simple two-degrees of freedom scheme is implemented; one part of the controller is devoted to the attenuation of load disturbances, and the other to the set-point following. The idea, in a few words, is simply that b has

to be increased when the convergence of the process output $y(t)$ to $y_{sp}(t)$ has to be speeded up, and decreased when the divergence trend of $y(t)$ from $y_{sp}(t)$ has to be slowed down. For the sake of simplicity, the methodology is implemented in such a way that the output of the fuzzy module is added to a constant parameter w , resulting in a coefficient $b(t)$ that multiplies the set-point.

2.A. Comparison between the fuzzy PID tuning methods

Based on extensive investigations [16], it is shown that the only two techniques that are generally able to improve the performances achieved with the simple use of a fixed set-point weight are the IFE and the FSW. In particular, the FSW appears to be the best method and it gives better step responses than in a standard nonlinear PID. This is because with the FSW method, both the overshoot and the rise time can be lower than the Ziegler-Nichols response. Conversely, results show that the SSP and FGS methods are not always worth using, as, in general, the same performances can be attained by simply applying a fixed set-point weight, simpler to determine and implement. On the other hand, for the IFE method, the choice of parameter k_3 is critical since its value has to be kept very low, otherwise the overall control system can be unstable. The derivative action is critical in the FGS scheme as well, while SSP control structure is very difficult to set, as it depends on the results obtained from the genetic algorithm, which is not always able to improve on the Ziegler-Nichols response and, when it succeeds, the selected rules are very difficult to interpret. The only scheme in which the setting of the fuzzy module's parameters is straightforward is the FSW one, for which a manual tuning procedure can be applied. Therefore, the FSW technique appears superior to the others, as it guarantees in general very good performances in the set-point and load disturbance step responses and it requires a modest implementation effort; hence its practical implementation in industrial environments appears to be very promising.

3. Fractional order $PI^\lambda D^\delta$ controllers

Fractional order control systems are described by fractional order differential equations. Fractional calculus allows the derivatives and integrals to be arbitrary order. The FOPID controller is the expansion result of the conventional PID controller based on fractional calculus. FOPID controllers

have five design parameters and improve the design flexibility.

3.A. Fractional calculus

There are several definitions of fractional derivatives. Perhaps the best known is the Grunwald-Letnikov definition. The m^{th} order fractional derivative of continuous function $f(t)$ is given by

$$\begin{aligned} {}_a D_t^m f(t) &= \lim_{h \rightarrow 0} h^{-m} \sum_{j=0}^{\left[\frac{t-m}{h}\right]} (-1)^j \binom{m}{j} f(t-jh) \\ &= \frac{d^m f(t)}{dt^m}, \end{aligned} \quad (3.1)$$

where $\left[\frac{t-m}{h}\right]$ is a truncation; $\binom{m}{j}$ are binomial coefficients, $\binom{m}{0} = 1$, $\binom{m}{j} = \frac{m(m-1)\dots(m-j+1)}{j!}$, that can be replaced by Gamma functions in a more general case. Thus, the fractional calculus operator (including fractional order and integer order differentiation or integration) is defined as

$${}_a D_t^\alpha = \begin{cases} d^\alpha/dt^\alpha & \Re(\alpha) > 0 \\ 1 & \Re(\alpha) = 0 \\ \int_a^t (d\tau)^{-\alpha} & \Re(\alpha) < 0. \end{cases} \quad (3.2)$$

The Laplace transform of the derivative of $f(t)$ is given by

$$L\{{}_0 D_t^\alpha f(t)\} = s^\alpha F(s) - \left[{}_0 D_t^{\alpha-1} f(t)\right]_{t=0}, \quad (3.3)$$

where $F(s)$ is the Laplace transform of $f(t)$. The Laplace transform of the integral of $f(t)$ is given by

$$L\left\{{}_0 D_t^{-\alpha} f(t)\right\} = s^{-\alpha} F(s). \quad (3.4)$$

3.B. Fractional order controllers

The differential equation of the $PI^\lambda D^\delta$ controller is described by

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\delta e(t). \quad (3.5)$$

The continuous transfer function of FOPID is obtained through the Laplace transform, which is given by

$$G_c(s) = K_p + K_i s^{-\lambda} + K_d s^\delta. \quad (3.6)$$

It is obvious that the FOPID controller not only needs a designing algorithm for three parameters, K_p , K_i and K_d , but also needs a designing algorithm for two orders of integral and derivative controllers (see [20]). The orders λ , δ are not necessarily integer, but any real numbers.

3.C. FOPID tuning methods

Several methods were suggested in order to tune an FOPID controller. Some of these methods are discussed in this section.

- *Minimization based method.*

This method which was proposed by Monje [7], is based on the desired behavior of the control system. As the FOPID controller has 5 degrees of freedom, we can have 5 constraints in order to tune the controller:

- 1) $|C(\omega_{cg})G(\omega_{cg})| = 0\text{dB}$, this constraint guarantees the gain crossover frequency of the open loop system.
- 2) $-\pi + \phi_m = \arg [C(\omega_{cg})G(\omega_{cg})]$, this constraint set the system's phase margin to a specific predetermined value.
- 3) $\left| \frac{C(\omega_h)G(\omega_h)}{1+C(\omega_h)G(\omega_h)} \right| < H$, this constraint is used in order to attenuate high frequency noises.
- 4) $\left| \frac{1}{1+C(\omega_l)G(\omega_l)} \right| < N$, this constraint use the sensitivity function in order to guarantee the output disturbance rejection and fine set point tuning.
- 5) $\frac{d}{d\omega} \arg [C(\omega)G(\omega)]|_{\omega=\omega_{cg}} = 0$, thus constraint is used for the robustness of the system against gain variations.

Now, an optimization method might be used in order to solve the above inequalities to find the optimal controller. In [11], a direct search simplex method was used.

- *Tuning based on parameter regulation rules (S shaped response).*

These rules can be applied to a system with an S shaped response to a unit step input. In these systems' response, Delay (L) and Time Constant (T) can be found. The parameters of the controller can be tuned then by the use of an appropriate look-up table. This table was found by solving

the previous inequalities by the following values:

$$\begin{aligned}\omega_{cg} &= 0.5\text{rad/s} \\ \phi_m &= 2/3\text{rad} \approx 38^\circ \\ \omega_h &= 10\text{rad/s} \\ \omega_l &= 0.01\text{rad/s} \\ H &= -10\text{dB} \\ N &= -20\text{dB}.\end{aligned}$$

- *Parameter regulation based on the critical gain.*

Valerio [8] proposed a parameter regulation method based on the critical gain of the system. Whenever system's gain increases up to a specific value, its step response will have oscillation. In this situation, system's gain and the oscillation's period determine the parameters which are used in the tuning rules. The parameters of the controller can be tuned then by the use of an appropriate look-up table. This table was found by solving the pre-mentioned inequalities by the above values.

- *FOPID tuning based on evolutionary techniques.*

Several evolutionary techniques were used in order to tune FOPID controllers. The most widespread technique is Genetic Algorithms. Usually GA method is used in order to minimize:

$$J = \int_0^{\infty} (w_1 |e(t)| + w_2 u^2(t)) dt, \quad (3.7)$$

where w_1 and w_2 can be determined by the designer.

4. Combination of FSW tuning method and FOPID controllers

It was shown that applying FSW tuning algorithm can lead to better response in time domain. On the other hand, FOPID controllers have significant advantages in terms of robustness to external disturbance, improvement in frequency response, and time response specifications. In this section, the incorporation of these two concepts is introduced, in order to use both FOPID controllers and FSW tuning method capabilities for reaching the best time response scheme. Accordingly, it is enough to do some changes in PID FSW tuning scheme.

Accordingly, once FOPID coefficients will fix corresponding with a mentioned standard FOPID tuning method and then the proportional coefficient updates through a fuzzy inference system, Figure 2 illustrates control

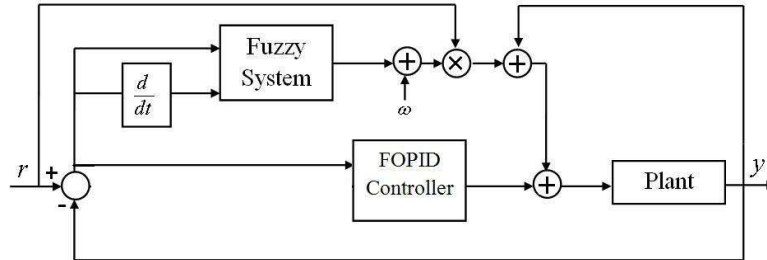


Figure 2: FFOPID based on FSW tuning algorithm scheme

scheme of the proposed fuzzy FOPID; similar to FSW scheme, the output of fuzzy module is added to a constant parameter (ω).

The fuzzy system which is used here has two inputs which are error of the output of the plant from the set point and rate of error changes. It also has one output which indicates the rate of controller parameters changes. In order to define the rule base of the fuzzy system, one of the common rule bases in fuzzy controllers is used here. It is called "Macvicar-Whelan". By the means of this predefined rule base the parameters of the FOPID controller can be tuned better. The following linguistic codes are used to indicate the inputs' and output's membership functions; -2:Negative Big, -1 :Negative medium, -0 :Negative Small, 0 :Small, +0 :Positive Small, +1 :Positive Medium, +2 :Positive Big.

Table 1 shows the rule base of the system. For instance one of the rules of the system is given here: IF error IS "Negative Medium" AND rate of error changes IS "Positive Small" THEN output IS "Small".

Table 1 clearly shows that the system has 6 MFs for each input and 5 MFs for the output. Therefore, the number of the rules is 36.

	-2	-1	-0	+0	+1	-2
-2	-2	-2	-1	-1	0	0
-1	-2	-1	-1	0	0	0
-0	-1	-1	0	0	0	1
+0	-1	0	0	0	1	1
1	0	0	0	1	1	2
2	0	0	1	1	2	2

Table 1: FFOPID control matrix

4.A. Optimization

Apparently, the response of the fuzzy FOPID controller completely depends on the membership functions. In order to find the best membership functions we use PSO optimization technique. However, first we need a performance criterion to achieve the desire specifications. Obtaining the robust stability is achievable using the H controller design. But this controller does not guarantee the good transient response. So in the cost function we confront the H norm as a constraint and define our cost function as a H_2 norm of the error. The overall performance criterion to achieve the desired specifications is as in equation (12) of [17],

$$\begin{aligned}
 \min J(\theta) &= \int_0^{\infty} e^T(t)e(t)dt \\
 s.t. & \\
 &\text{nominal closed loop has to be stable,} \\
 &g_{\infty}(\theta) - 1 < 0,
 \end{aligned} \tag{4.1}$$

with $g_{\infty}(\theta) = \sqrt{g_1(\theta)^2 + g_2(\theta)^2}$ and $g_1(\theta)$ is defined in order to obtain the robustness of the system in high frequencies. So the condition in inequality (4.2) must be satisfied:

$$g_1(\theta) = \|W_1(s)T(s)\|_{\infty} < 1, \tag{4.2}$$

where $T(s)$ is the complementary sensitivity function of the system, which can be calculated by the means of

$$T(s) = K(s)G(s)(I + K(s)G(s))^{-1}. \tag{4.3}$$

And $g_2(\theta)$ is also defined in order to attenuate the effect of low frequency disturbances. So the condition in inequality (4.4) must be satisfied,

$$g_2(\theta) = \|W_2(s)S(s)\|_\infty < 1, \quad (4.4)$$

where $S(s)$ is the sensitivity function of the system, which can be calculated by the means of

$$S(s) = (I + K(s)G(s))^{-1}. \quad (4.5)$$

Also to ensure the stability, we penalize the cost function with a penalty function which is obtained according to the equation (4.6),

$$P(\theta) = \begin{cases} P_1; & \text{if } \theta \text{ is unstable.} \\ 0; & \text{else.} \end{cases} \quad (4.6)$$

If the individual does not satisfy the stability then it is an unstable individual and it is penalized with a very large positive constant. Parameters vector, θ , is the membership functions values of the fuzzy system. For example the membership functions for the first input are as shown in Fig. 3; θ is defined such as the structure of the membership functions remains unchanged. Thus the centers of the membership functions are considered to be fixed. Fig. 3 shows an example of a triangular membership function and its variable parameters. Note that a and b are positive numbers. The parameters vector is defined for all the membership functions in the fuzzy system, similarly. For instance, if the fuzzy system has 6 MFs (like Fig 3) for each input and 5 MFs for output, θ contains $10+10+8=28$ parameters.

As normal PSO is an evolutionary method and it cannot solve the mentioned constrained optimization problem directly, the next step is to convert the constrained optimization problem into non-constrained one. So the new problem is introduced as the minimization of

$$F(\theta) = \int_0^\infty e^T(t)e(t)dt \times \delta(\phi(\theta) + P(\theta)), \quad (4.7)$$

where δ is the penalty factor which regulates the speed and precision of the convergence. And $\phi(\theta)$ is the penalty for the $g_\infty(\theta) - 1 < 0$ constraint; $\phi(\theta)$ is defined as $\phi(\theta) = (g^+(\theta))^2$. where $g^+(\theta) = \max(0, g_\infty(\theta) - 1)$. In other words, $g^+(\theta)$ indicates the magnitude of the violation of the second constraint of the problem. The final step of the design procedure is optimizing the cost function by the use of an optimization method. As mentioned

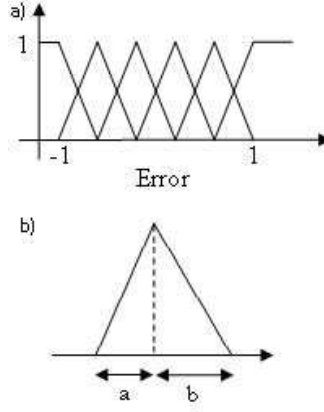


Figure 3: a)Error Membership Functions,b)Membership Function Variables.

earlier the PSO method is used in order to find a solution for the optimal controller design.

The PSO method updates the values of each particle in each population by the use of equations (4.8) and (4.9).

$$v_{id}(t+1) = w \cdot v_{id}(t) + c_1 \cdot rand() \cdot (p_{id} - x_{id}(t)) + c_2 \cdot rand() \cdot (p_{gd} - x_{id}(t)). \quad (4.8)$$

$$x_{id}(t+1) = (1 - mc) \cdot x_{id}(t) + mc \cdot v_{id}(t), \quad (4.9)$$

where $X_i = (x_{i1}, \dots, x_{iD})$ indicates the i -th particle vector, $P_i = (p_{i1}, \dots, p_{iD})$ indicates the best coordinates of the i -th particle, $V_i = (v_{i1}, \dots, v_{iD})$ indicates the velocity of the i -th particle and indicates the coordinate vector of the best particle (see [18]). Also w , c_1 , c_2 and mc are the inertia weight, acceleration constants and torque coefficient, respectively. The inertia weight can be also updated in the each iteration by

$$w = (w_{\max} - w_{\min}) \times \frac{(Iter_{\max} - Iter_{now})}{Iter_{\max}} + w_{\min}. \quad (4.10)$$

5. Simulation results

In this section, the performance of different FOPID controller based on FSW tuning (FFOPID) method has investigated to verify their time

response specifications. Here we introduce simulations which demonstrate effectiveness of the proposed controller over present FOPID tuning methods. To that end, we take into account five different type transfer functions. The controllers have designed in different FOPID tuning methods. The results are compared with proposed FFOPID. For simulation, we use of Oustaloup et al method to approximate fractional controllers with usual (integer order) transform functions. This method is fully described in [19]. First, consider a first-order transfer function with a delay as

$$G_1 = \frac{0.2e^{-s}}{62s + 1}. \quad (5.1)$$

Based on FOPID tuning which is presented in [7], one can achieve following fractional order controller

$$C_1(s) = 7.9619 + \frac{0.2299}{s^{0.9646}} + 0.1504s^{0.0150}. \quad (5.2)$$

Fig. 4 shows closed loop step response of the FOPID, FFOPID and classic Z-N controllers. The FOPID based on FSW tuning (FFOPID) clearly have better time specifications. Next investigated transfer function is a second-order transfer function. It is

$$G_2 = \frac{1}{0.7414s^2 + 0.2313s + 1}. \quad (5.3)$$

The controller is designed in (5.4), $C_2(s)$, according to demonstrated method in [9],

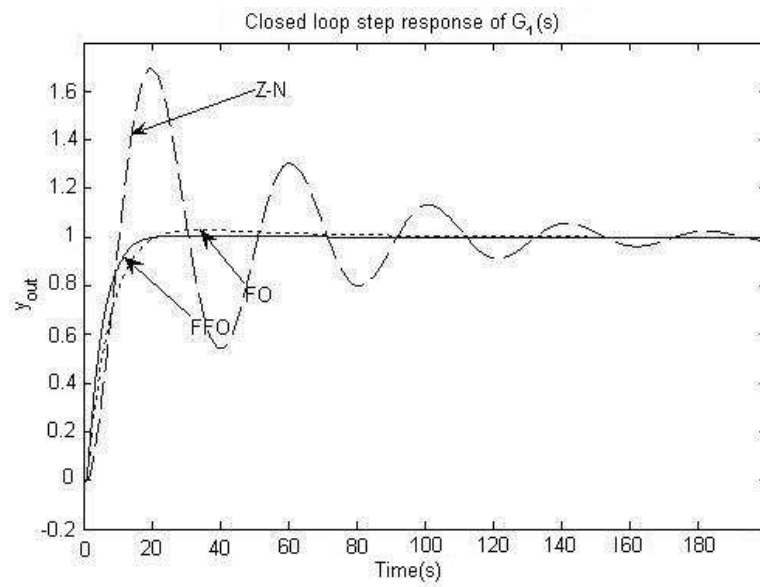
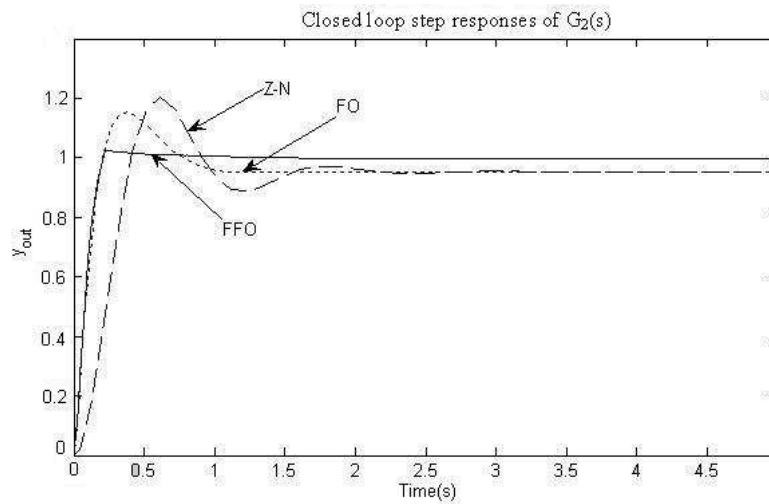
$$C_2(s) = 20.5 + 2.7343s. \quad (5.4)$$

To compare time specifications of FFOPID, FOPID and classic Z-N, their step response is shown in Fig. 5. As it observes, overshoot, rise time and settling time of FFOPID is much better than other methods. Third transfer function $G_3(s)$ is second-order transfer function without pole on the zero. It is

$$G_3 = \frac{1}{4.3200s^2 + 19.1801s + 1}. \quad (5.5)$$

With utilizing the FOPID tuning according to [10] the controllers, C_3 , will be

$$C_3(s) = 6.9928 + \frac{12.4044}{s^{0.6000}} + 4.1066s^{0.7805}. \quad (5.6)$$

Figure 4: Closed loop step responses of $G_1(s)$.Figure 5: Closed loop step responses of $G_2(s)$.

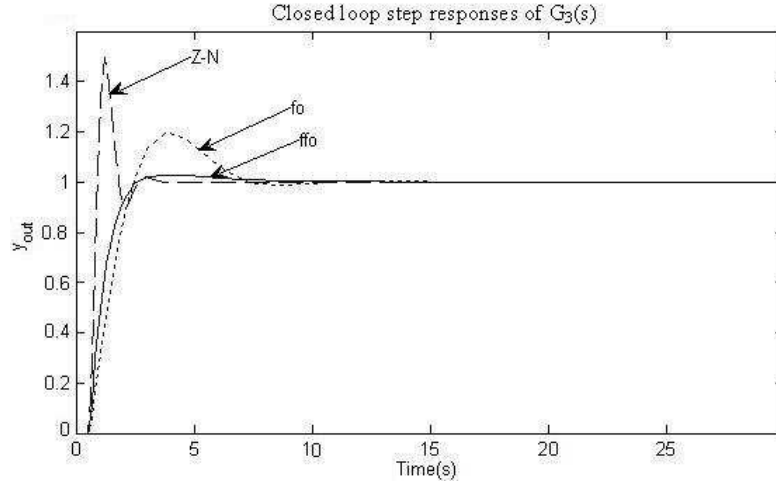
Figure 6: Closed loop step responses of $G_3(s)$.

Fig. 6 reveals advantages of our proposed FFOPID over FOPID and classic PID tuning. Consider $G_4(s)$ as a third-order plant transfer function

$$G_4 = \frac{1}{s^3 + 2.539s^2 + 62.15s}. \quad (5.7)$$

According to the FOPID tuning controller method in [8], the controller $C_4(s)$ has following transfer function.

$$C_4(s) = 0.8271 + \frac{14.3683}{s^{0.5588}} - 1.6866s^{1.2328}. \quad (5.8)$$

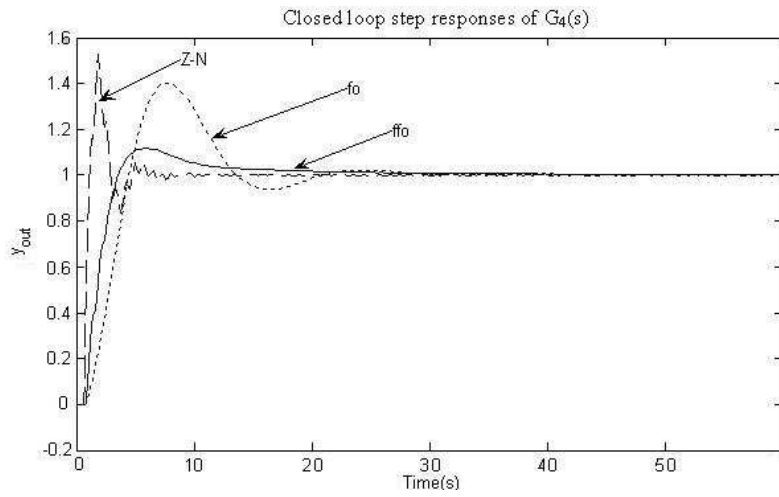
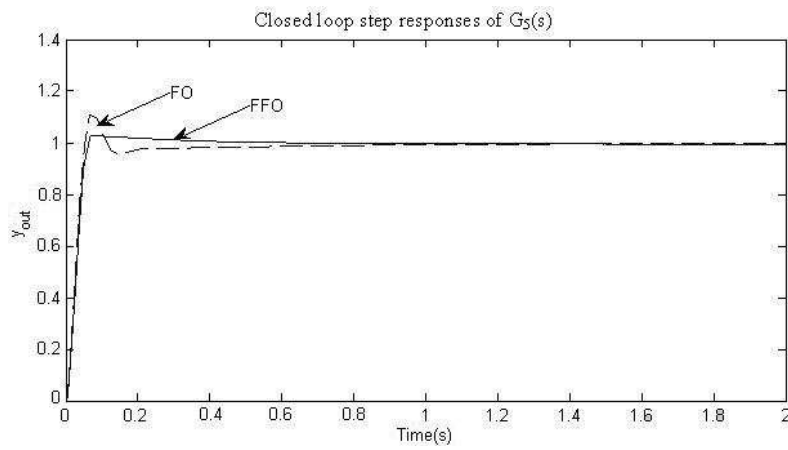
Step responses in Fig. 7 describe FFOPID significant preference to other methods. Finally, consider

$$G_5 = \frac{400}{s^2 + 50s}. \quad (5.9)$$

and

$$C_5(s) = 2.86 + \frac{0.000012}{s^{0.00119}} + s^{0.4896}. \quad (5.10)$$

As a controller which is designed based on the PSO optimization method, see [11]. Step response of FOPID and proposed FFOPID are shown in Fig. 8. Although the rise time and settling time of them is pretty equal, the system with FFOPID controller has less overshoot.

Figure 7: Closed loop step responses of $G_4(s)$.Figure 8: Closed loop step responses of $G_5(s)$.

6. Conclusions

This paper presents a novel structure for a controller, which is called Fuzzy FOPID controller, to enhance system performance. The proposed method for optimizing the controller involves both time-domain and frequency-domain performance criteria. Application of the method on different types of systems shows that the proposed controller can act well enough in these systems. Furthermore, it can be concluded from the simulations that the proposed FFOPID controller has better robust stability and performance characteristics than the FOPID and PID controllers applied to the same systems.

References

- [1] I. Petras, L. Dorcak, and I. Kostial, Control quality enhancement by fractional order controllers. *Acta Montanistica Slovaca* **3**, No 2 (1998), 143-148.
- [2] C.B. Ma and Y. Hori, Backlash vibration suppression in tensional system based on the fractional order Q-filter of disturbance observer. In: *Proc. 8th IEEE Int. Workshop on Advanced Motion Control*, 2004, 577-582.
- [3] N.M.F. Ferreira and J.A.T. Machado, Fractional-order hybrid control of robotic manipulators. In: *Proc. of the 11th International Conf. on Advanced Robotics*, Coimbra, Portugal, June-July 2003, 393-398.
- [4] A. Kailil, N. Mrani, M.M. Touati, S. Choukri, and N. Elalami, Low earth-orbit satellite attitude stabilization with fractional regulators. *International Journal of Systems Science* **35**, No 10 (Aug. 2004), 559-568.
- [5] R. Caponetto, L. Fortuna and D. Porto, Parameter tuning of a non integer order PID controller. In: *Electronic Proc. of the 15th Intern. Symposium on Mathematical Theory of Networks and Systems*, 2002.
- [6] R. Caponetto, L. Fortuna and D. Porto, A new tuning strategy for a non integer order PID controller. In: *1st IFAC Workshop on Fractional Differentiation and its Applications*, Bordeaux, France, July 2004.
- [7] C.A. Monje, B.M. Vinagre, Y.Q. Chen, V. Feliu, P. Lanusse and J. Sabatier, Proposals for fractional PID tuning. In: *1st IFAC Workshop on Fractional Differentiation and its Applications*, Bordeaux, France, July 2004.
- [8] D. Valério, J.M.G. da Costa, Tuning-rules for fractional PID controllers. In: *2nd IFAC Workshop on Fractional Differentiation and its Applications*, Porto, Portugal, July 2006.

- [9] D. Valério, J.M.G. da Costa, Tuning of fractional PID controllers with Ziegler-Nichols type rules. *Signal Processing* **86**, No 10 (2006), 2771-2784.
- [10] I. Petras, L. Dorcak, I. Kostial, Design of fractional-order controllers with given stability and dumping measures. In: *9th Intern. BERG Conference, Kosice, Slovak Republic*, 1997, 116-119.
- [11] J. Cao and B. Cao, Design of fractional order controller based on particle swarm optimization. *Intern. J. of Control, Automation, and Systems* **4**, No 6 (2006), 775-781.
- [12] S.G. Tzafestas, and N.P. Papanikolopoulos, Incremental fuzzy expert PID control. *IEEE Trans. Ind. Electron.* **37**, No 5 (1990), 365-371.
- [13] S.-Z. He, S. Tan , and F.-L. Xu, Fuzzy self-tuning of PID controllers. *Fuzzy Sets Sys.* (1993), 37-46.
- [14] Z.-Y. Zhao, M. Tomizuka, and S. Isaka, Fuzzy gain scheduling of PID controllers. *IEEE Trans. Syst. Man, Cybern.* **23**, (1993), 1392-1398.
- [15] A. Visioli, Fuzzy logic based set-point weighting for PID controllers. *IEEE Trans. Syst. Man, Cybern.* **29** (1999), 587-592.
- [16] A. Visioli, Tuning of PID controllers with fuzzy logic. *IEE Proc. Control Theory Appl.* **148**, No 1 (2001), 1-8.
- [17] M. Zamani, M.K. Ghartemani, N. Sadati, Desing of an Hinf-optimal FOPID Controller Using Particle Swarm Optimization. In: *Proc. of IEEE Chinese Control Conference, China*, 2007.
- [18] M. Zamani, M.K. Ghartemani, N. Sadati, FOPID Controller Design for robust Performance Using Particle Swarm Optimization. *Fract. Calc. Appl. Anal.* **10**, No 2 (2007), 169-188.
- [19] A. Oustaloup, F. Levzon, B. Mathieu, and F.M. Nanot, Frequency-band complex non integer differentiators: Characterization and synthesis. *IEEE Trans. on Circuits and Systems I: Fundamental Theory and Applications* **1** (2000), 25-39.
- [20] I. Podlubny, Fractional-order systems and PID-controllers. *IEEE Transaction on Automatic Control*, **44**, No 1 (1999), 208-214.

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Received: May 11, 2009

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